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Study the BER Performance Comparison of MIMO Systems Using BPSK Modulation with ZF and MMSE Equalization

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ABSTRACT

In the modern age, wireless communication is very helpful in various mobile antenna communication systems. In mobile communication systems, the transmission of data transfer rates is very high and it plays an important role in several services like video, top-quality audio, and mobile integrated service digital network. During the transmission of data at higher data transmission rates through the mobile radio channels, the channel impulse response can spread over many symbol periods as well as causes inter-symbol interference (ISI). Wireless transmission is suffering from fading and interference effects which may be combated with equalizer. Due to fading and interference, it creates a problem for signal recovery in wireless communication. The main objective of this paper is to analyze the different types of equalizers such as ZF and MMSE for BPSK modulation. The simulation result has been developed by using MATLAB toolbox version 2015a and a multi-tap ISI channel is considered. By analyzing the simulation result it shows that if the number of tap lengths is increasing, BER will decrease in ZF equalizer. And finally shows BER vs SNR comparison of two different types of an equalizer and is able to find out MMSE performance is better than ZF equalizer.

Keywords: MIMO systems, BPSK modulation, BER performance, ZF, MMSE equalization, AWGN, and SNR.

1. INTRODUCTION

We are currently living in the age of wireless technology and day by day the use of wireless technology is growing at a tremendous rate. Multiple Input Multiple Output (MIMO) refers to the antenna configuration technology for wireless transmissions where the multiple antennas are used at both transmitter and receiver sides. The antennas at both ends of the communications circuit are combined to decrease errors and optimize data speed (David *et al.*, 2003; Zhang and Kung, 2003). The MIMO system uses

multiple antennas to transmit multiple parallel signals for transmission. The MIMO techniques have attracted great attention due to multiple antennas at both transmitter and receiver being employed; it provides the possibility of upper data rates compared to single antenna systems (Telatar, 1999; Tarokh *et al.*, 1999). A point-to-point (for single user) MIMO communication system promises large gains for both data transmission rates and reliability and these are accomplished via the utilization of space-time codes (diversity gain oriented) (Ju *et al.*, 2013).

This technique grants higher channel capacity to wireless systems and is able to increase capacity of the channel linearly with the numerous antennas and link range without additional bandwidth and power requirements (Trimeche *et al* 2012).

ISI is a one kind of problem in high speed communication. Sometimes ISI can't work perfectly when a transmission interferes with itself and the receiver can't decode the transmission correctly (Theodore, 2002). All types of assumptions consist of AWGN and this type of channel model is not common in the real world. For the lack of frequency spectrum we usually filter the transmitted signal to limit its bandwidth so that we can achieve efficient sharing of frequency response of the channel. Since various workable channels are band-pass and actually, they often respond differently to inputs with different frequency components, i.e. they are dispersive. We need to refine the simple AWGN (or non-dispersive) model so that we can represent this type of practical channel accurately. We only consider the refinement of the dispersive channel mode is,

$$r(t) = u(t) \otimes h_c(t) + n(t) \quad (1)$$

Where, $u(t)$ is the transmitted signal, $h_c(t)$ is the impulse response of the channel, and $n(t)$ is AWGN with power spectral density $\frac{N_0}{2}$.

Wireless communication systems have great advantages and provide higher data rates, Quality of Service (QoS), and link reliability (Ananth, 2012). The modern wireless transmission system has some applications such as IEEE 802.11n (Wi-Fi), 4G, 3GPP Long Term Evolution and WiMAX. The MIMO system uses Multiple Transmit and Multiple Receive antennas which take advantages of multipath propagation during a high distraction environment.

The main objective of this present study is to analyze the different types of equalizers such as ZF and MMSE for BPSK modulation. Then also shows BER vs SNR comparison of two different types of equalizer and try to find out which is the best equalizer of BER for a given SNR (Kumar and Kumar, 2011; Jagan *et al.*, 2010). This paper is designed as follows: Section 2 explains the channel model. Section 3 explains BPSK

technique. Section 4 interprets ZF equalizer. Section 5 interprets MMSE equalizer. Section 6 is a simulation model. Section 7 describes simulation results and discussion. And finally the conclusion is drawn in section 8.

2. CHANNEL

Additive White Gaussian Noise is the basic noise model that is used in communication channels for thermal noise. It has some basic ideas:

Additive: the noise is additive, i.e., the received signal is equal to the transmit signal by adding some noise, where the noise is linearly independent of the signal of the channel.

White: the noise is white, i.e., the ability spectral density is flat, therefore the autocorrelation of the noise in time domain is zero for any non-zero time offset.

Gaussian: the noise samples have a normal distribution and the average time domain value is zero. AWGN is usually employed in channel models which are capable of communicating a linear addition of wideband or white noise with a relentless spectral density (expressed as watts per hertz of bandwidth) and therefore the amplitude is Gaussian distribution. The model does not calculate for fading, frequency selectivity, interference and dispersion. Fading is the time variation of the received signal with various variables. However, it produces some simple and obedient mathematical models which are useful for several satellite and deep space communication links. Wideband noise has some natural noise resources: they're thermal variations of atoms in conductors (is called thermal noise), short noise and from Godlike sources like the sun.

a. Transmit Symbol

Let the transmit symbols be modeled as

$$S(t) = \sum_{n=-\infty}^{\infty} a_n g(t - mT) \quad (2)$$

Where

T is the symbol period, a_n is the symbol to transmit, $g(t)$ is the transmit filter, n is the symbol index and $S(t)$ is the output waveform.

Now, let us assume that the transmit pulse shaping filter is not present, i.e, so the transmit symbols can be modeled by the discrete time equivalent

$$s[k] = a_n$$

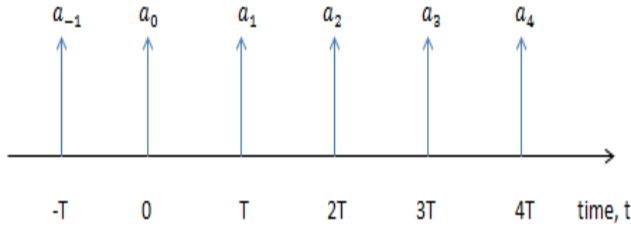


Fig 1: Transmit Symbols.

b. Channel Model

Consider 3-tap multipath channel with spacing T i.e.

$$h[k] = [h_1 h_2 h_3]$$

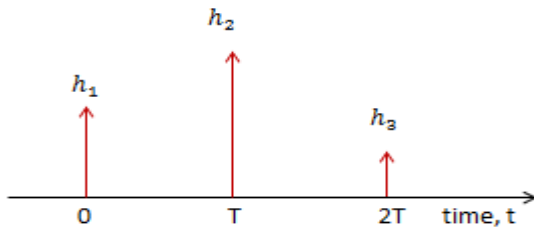


Fig 2: Channel Model (3 tap multipath).

When the received signal falls into corrupted by noise n as a result of adding a multipath channel then it is known as Additive White Gaussian Noise (AWGN). The value of the noise n follows the Gaussian probability distribution function

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

With mean $\mu = 0$ and variance $\sigma^2 = \frac{N_0}{2}$.

The received signal is $y[k] = s[k] \otimes h[k] + n$ (4)

Where, \otimes is the convolution operator (Gupta et al., 2011).

3. BINARY PHASE SHIFT KEYING

BPSK is a one kind of modulation technique which has two phases and it is the simplest form of PSK. It is referred to as 2-PSK. This two phases are separated by 180° and represented this two phases are 0° and 180° .

Where, $\theta = 0^\circ$ represents for binary message 1, and $\theta = 180^\circ$ represents for binary message 0

a. Implementation

The general form of BPSK modulation follows the equation

$$S_n(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi ft + \pi(1 - n)) , n = 0, 1 \tag{5}$$

The two phases are 0° , and 180° . These two phase represents two carrier signals and this are given below

$$S_0(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi ft + \pi)$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi ft) \text{ for binary "0"} \tag{6}$$

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi ft) \text{ for binary "1"} \tag{7}$$

Where, f is the frequency of the base band, E_b is the energy per bit, and T_b is the bit duration.

b. Bit Error Rate

The bit error rate (BER) of BPSK under Additive White Gaussian Noise (AWGN) can be calculated as

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{8}$$

This is often also the symbol error rate because it represents only one bit per symbol.

4. ZERO FORCING EQUALIZER

Zero Forcing equalizer is a method that applies inversely to the received signal so that it can restore the signal. It is a one kind of graphical equalization algorithm that is widely used in communication systems. This form of equalizer was first proposed by Robert Lucky.

It has many useful applications. For example, by studying deeply of the MIMO system in 802.11n to know the accurate information about the channel at every antenna and two or more streams are recovered from the receive signal. By applying ZF equalizer technique it will be possible to put down the value of Inter Symbol Interference (ISI) at zero for a noise free channel (Islam et al., 2020). This will be useful when

ISI is significant compared to noise. The Zero Forcing Equalizer $C(f)$ is represented by $C(f) = 1/F(f)$ where f is a channel frequency response. The combination of channel and equalizer presents a flat frequency response and linear phase $F(f)C(f) = 1$ (Ju et al., 2013; Kumar and Piyush, 2017)

a. Zero Forcing(ZF) Equalizer for 2x2 MIMO channel

Consider a 2x2 MIMO channel, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (9)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (10)$$

Where,

y_1, y_2 are the received symbol on the first and second antenna respectively,

$h_{1,1}$ is the channel where 1 is the both 1st sending antenna and 1st receive antenna,

$h_{1,2}$ is the channel where 2 is the 2nd sending antenna and 1 is the 1st receive antenna,

$h_{2,1}$ is the channel where 1 is the 1st sending antenna and 2 is the 2nd receive antenna,

$h_{2,2}$ is the channel where 2 is the both 2nd transmit antenna and 2nd receive antenna,

x_1, x_2 are the sending symbols, and

n_1, n_2 is the noise on 1st, 2nd receive antennas.

The equation can be illustrated in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (11)$$

Equivalently,

$$Y = HX + N \quad (12)$$

Where,

Y = Received Symbol Matrix, H = Channel matrix, X = Transmitted symbol Matrix, and N = Noise Matrix.

To solve for x , we need to find a matrix W which satisfies $WH = I$. The Zero Forcing (ZF) detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H \quad (13)$$

Where,

W - Equalization Matrix, and H - Channel Matrix.

This matrix is known as the Pseudo inverse for a general $m \times n$ matrix where

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad (14)$$

Furthermore, it can be seen that, following zero forcing equalization, the channel for symbol transmitted from each spatial dimension (space is antenna) is a like a 1x1 Rayleigh fading channel. Hence the BER for 2x2 and 4x4 MIMO channel in Rayleigh fading with Zero Forcing equalization is similar to the BER obtained for a 1x1 channel in Rayleigh fading (Prasad and Varanasi, 2004).

For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \frac{1}{2} \left(1 - \sqrt{\frac{\frac{E_b}{N_o}}{(\frac{E_b}{N_o}) + 1}} \right) \quad (15)$$

Where,

P_b - Bit Error Rate, and $\frac{E_b}{N_o}$ - Signal to noise Ratio.

5. MINIMUM MEAN SQUARE ERROR EQUALIZER

A Minimum Mean Square Error (MMSE) is a conventional way that calculates the Mean Square Error (MSE) and tries to minimize the error. For this reason it refers to the best common measure estimator quality. The main property of MMSE is that it fails to remove all ISI perfectly but decreases the total power of the noise and ISI components in the output. Let x be an unknown random variable, and let y be a known random variable. An estimator $\hat{x}(y)$ is any function of the measurement y , and its mean square error is given by

$$MSE = E \{ (X - \hat{X})^2 \} \quad (16)$$

Where; the expectation is taken over both x , and y .

By following the term $AY + b$ we acquire a minimum MSE overall estimator and it is known as linear MMSE estimator. Where Y is a random vector, A is a matrix and b is a vector.

a. Minimum Mean Square Error Equalizer for 2x2 MIMO channel

Consider a 2x2 MIMO channel, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \ h_{1,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \quad (17)$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \ h_{2,2}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \quad (18)$$

Where,

y_1, y_2 are the received symbol on the first and second antenna respectively,

$h_{1,1}$ is the channel where 1 is the both 1st sending antenna and 1st receive antenna,

$h_{1,2}$ is the channel where 2 is the 2nd sending antenna and 1 is the 1st receive antenna,

$h_{2,1}$ is the channel where 1 is the 1st sending antenna and 2 is the 2nd receive antenna,

$h_{2,2}$ is the channel where 2 is the both 2nd sending antenna and 2nd receive antenna,

x_1, x_2 are the sending symbols, and

n_1, n_2 is the noise on 1st, 2nd receive antennas.

The equation can be illustrated in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \quad (19)$$

Equivalently,

$$Y = HX + N \quad (20)$$

Where,

Y = Received Symbol Matrix, H = Channel matrix, X = Transmitted symbol Matrix, and N = Noise Matrix.

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the criterion,

$$E \{ [W_{y-x}] [W_{y-x}]^H \} \quad (21)$$

Where,

W - Equalization Matrix, H - Channel Matrix, n - Channel noise, and y - Received signal.

To solve for x , we need to find a matrix W which satisfies $WH = I$. The Minimum Mean Square Error

(MMSE) detector for meeting this constraint is given by,

$$W = [H^H H + N_o I]^{-1} H^H \quad (22)$$

This matrix is known as the Pseudo inverse for a general m x n matrix where

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \quad (23)$$

When comparing two equations i.e. eq.(13) and eq.(22) in Zero Forcing Equalizer, we see that both equations are comparable except $N_o I$. Actually, when the value of the noise term is zero, the MMSE equalizer minimizes to Zero Forcing equalizer.

6. SIMULATION MODEL

The following steps were taken to design the system –

- (a) Random binary sequence is generated.
- (b) Applying BPSK modulation where bit 0 represented as -1 and bit 1 represented as +1.
- (c) Now convolving the symbol with a multipath channel.
- (d) White Gaussian Noise is added.
- (e) Now two different programs are prepared –

1. First zero forcing equalization technique is applied for varying tap length and BER is calculated (Islam and Hossain, 2019). Then simulated BER value is compared. Then demodulated the received data and count no. of bit errors for repeating values of $\frac{E_b}{N_o}$.
2. Second we performed MMSE equalization technique for different tap-lengths and calculated BER and compared BER values between two equalizers namely MMSE and ZF. Finally, simulated value is done.

7. RESULT AND DISCUSSION:

This section provides a summarized interpretation and comparative analysis for signal detection using BPSK modulation in ZF and MMSE equalizer. All simulation processes developed in the MATLAB environment.

As shown in Fig 1, this is the analysis for zero forcing equalizer for various tap lengths. From the graph it is noticed that if the number of tap lengths are increasing, the bit error rate will decrease. Increasing the equalizer tap length from 3 to 5 showed feasible performances developed but when the tap length is 9, then BER decreases than any other tap length. Furthermore, it is seen that there is a lot of gap between theoretical and practical BER.

a. Bit Error Rate probability curve for BPSK modulation in ISI with ZF equalizer

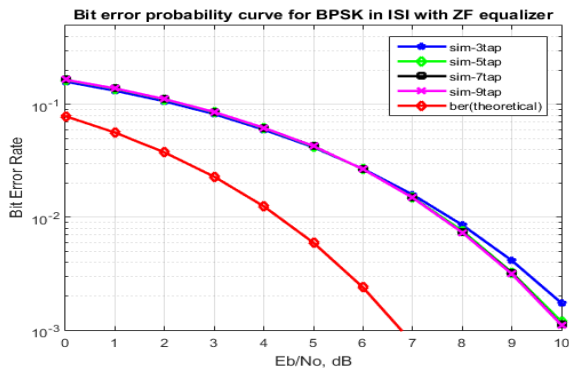


Fig 3: Bit error probability curve for BPSK in ISI with ZF equalizer.

b. Comparison BER curve between ZF and MMSE equalizer

This is the result of a comparison bit error probability curve between Zero Forcing and Minimum Mean Square Error equalizer as shown in Fig 2. It can be seen from Fig 2 that bit error rate value of MMSE has decreased than ZF equalizer for fixed tap length. But there is a gap between theoretical and practical BER. So MMSE detectors are more efficient than ZF.

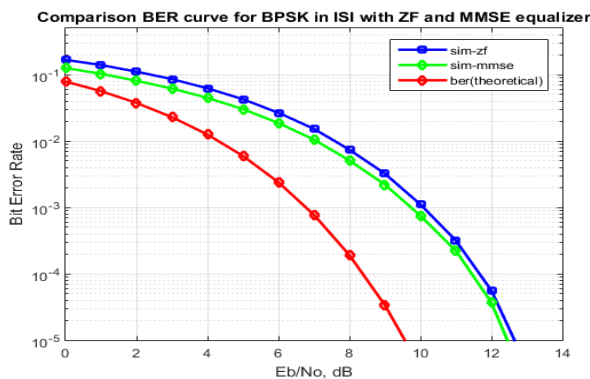


Fig 4: Comparison BER curve for BPSK modulation with ZF and MMSE equalizer.

c. Comparison BER values between ZF and MMSE equalizer using BPSK modulation

In the following Table 1 shows Bit Error Rate (BER) values comparison between ZF and MMSE equalizer. This table analysis shows that the simulated bit error rate (simBer) value of MMSE is much smaller than the ZF equalizer. When the SNR value is gradually increased, the BER value decreases and finally when the SNR value reaches 13, 14 and 15 then the BER value remains to 0.

Table 1: BER performance comparison between ZF and MMSE using BPSK modulation.

SNR (dB) Value	BER Value	
	simBer_ZF	simBer_MMSE
0	0.1664	0.1252
1	0.1383	0.1031
2	0.1113	0.0814
3	0.0855	0.0617
4	0.0623	0.0444
5	0.0423	0.0297
6	0.0264	0.0184
7	0.0148	0.0102
8	0.0075	0.0050
9	0.0031	0.0020
10	0.0011	0.0006
11	0.0003	0.0001
12	0.0001	0.0000
13	0.0000	0.0000
14	0	0
15	0	0

Based on the simulation results analyzed so far, it is concluded that MMSE is more accurate and suits a wide range of channel state conditions expressed in SNR ratio. The zero forcing method is an approximate method more suitable for high signal to noise ratio. The MMSE is working well for all signals to noise ratios but it needs huge computational effort.

8. CONCLUSION:

This paper presents the particular information about the Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) Equalizer as well as show the comparison of BER performance between them using

BPSK modulation. All the simulation results will be shown about the Bit Error Rate (BER) features for the both types of equalizers. This paper uses the MATLAB framework for the analysis of simulation. Based on the simulation results we learn that as the tap length increases, BER decreases with Zero Forcing (ZF) equalization for noise free channel. On the other hand, when we compare both equalization techniques it is seen that the BER value of MMSE is decreased for the AWGN channel than ZF equalizer. When SNR value is serially high the BER value gradually decreases and finally reaches the value is 0. ZF equalizer enhances the noise in the channel while the MMSE equalizer provides better noise immunity and removes a marginal noise. Furthermore, Zero Forcing and MMSE equalizers were simulated and the results compared. Finally, the results show that MMSE equalizer performs better than ZF equalizer.

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10. CONFLICT OF INTERESTS:

The author (s) declared no potential conflicts of the interest to publish it.

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